xCoAx 2019 Conference on Computation, Communication, Aesthetics & X Milan, Italy

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Between Chaotic Synthesis and Physical Modelling: Intrumentalizing with Gutter Synthesis

Keywords
chaotic synthesis
interaction
hysteresis
physical modelling
gutter synthesis

This paper presents the Gutter Synthesis project, a synthesis method that combines chaotic synthesis based on the Duffing Oscillator dynamical system with modal-like resonances. The synthesis process is described and the project is related both to prior work on chaotic synthesis and to relevant perspectives from physical modelling. A range of specific kinds of interactions with the synthesis engine are considered, supported by accompanying videos. These interactions demonstrate the complexity of behaviours that can be encountered when interacting with chaotic systems, particularly in relation to hysteresis. The potential of the system to enter different states, and for unstable boundary points to be explored as creative resources are considered and linked to Andy Keep's notion of *instrumentalizing*.

1 INTRODUCTION

Gutter synthesis is the name given to the specific synthesis process described in this paper, to the software,[1] and to a CD release of sonic explorations of the synthesis process (Mudd 2018). The project explores the overlap between the well-trodden fields of chaotic synthesis on the one hand and physical modelling on the other. The motivations behind research in the two domains can often differ, with physical modelling attempting to mimic, understand or expand on existing acoustic situations, and chaotic synthesis exploring very digital, artificial and potentially noisy systems. Nevertheless, both make frequent use of nonlinear dynamical systems, and exploit the potential for these systems to produce a variety of tones and timbres through fixed sets of rules. The sound world afforded by the Gutter Synthesis software is deliberately acoustically inspired, with the noisy proclivities of the chaotic synthesis processes constrained by sets of modal resonators that behave in a manner analogous to the bore in blown instruments or the string in bowed or struck string instruments.

This paper explores *interaction* in relation to the Gutter Synthesis software, and hence in relation to acoustically inspired chaotic synthesis processes. The complex nature of the system's behaviour in response to even relatively simple inputs permits a highly exploratory engagement, where the user may not always be sure how the instrument will respond. This is linked here to Andy Keep's notion of *instrumentalizing*: the process of exploring sonic objects and responding to their inherent (but often hidden) sonic properties (Keep 2009) This is as opposed to, for example, bringing a pre-formed musical language to the object and trying to govern the response of the object in relation to that pre-formed language. Keep connects this approach with experimental contemporary music, and in particular free improvisation, but it can also be linked to contemporary computer music practices (Mudd 2017). An important corollary of Keep's focus on the act of exploring objects and instruments as material is the emphasis that this puts on interaction. In the case of digital tools particularly, it is not straightforward to create software that has scope for nuanced exploration, and that has the potential for hidden elements to be uncovered and developed. Chaotic synthesis is explored here as a method for permitting these kinds of creative engagements in digital interactions, a point that is made clearer through the inherent link with physical modelling.

In order to address the nature of possible interactions with the Gutter Synthesis software, a series of short demonstration videos[2] accompany this paper, which provide examples of the specific kinds of interaction discussed, showing how the audio engine responds to particular changes in input. It is also recommended that the software is used alongside the reading of this paper to support a more thorough understanding of the discussion.

[1] The software is available at http://entracte.co.uk/projects/tom-mudd-e226/

[2] Available at https://vimeo.com/album/5707465

2 CHAOTIC SYNTHESIS AND PHYSICAL MODELLING

Chaotic synthesis has multiple roots, depending on how the field is defined. The cybernetic experiments of the 1960s, electrical feedback experiments of David Tudor (Kuivilla 2004), and perhaps more specifically the implementations of Rössler's equations as video and sound synthesis processes were developed and explored by a range of artists. A wide range of other dynamical systems have been applied in both digital and analogue settings for a variety of reasons. The use of such systems is often motivated by a desire to explore and exploit the complex, emergent behaviours that they exhibit: their ability to move between ordered oscillations and more unpredictable and turbulent states (Degazio 1993; Mackenzie 1995; Radunskaya 1996; Scipio 1990) and often their potential to structure not only micro-level timbral aspects, but also macro-level structural aspects (Pressing 1988; Scipio 1990).

Physical modelling has a parallel history, emerging from initial experiments with voice synthesis in the 1960s (Välimäki et al. 2006). McIntyre, Schumacher, and Woodhouse (1983) provide a highly simplified but useful characterisation of musical instruments as composing a nonlinear element coupled to a passive linear element, shown in Fig. 1. The former can represent the behaviours of reeds, bowed interactions, air-jet behaviour, and so on, while the latter can represent the response of a string or a tube.[3] The connection between chaotic synthesis and physical modelling has been made explicit by a number of authors (Dobson and Fitch 1995; Radunskaya 1996; Truax 1990). Although the two domains may start with very different motivations, they arrive at a similar place in terms of digital synthesis processes. Physical modelling synthesis algorithms are generally discrete renderings of nonlinear dynamical systems that are very similar to the kinds of systems explored through chaotic synthesis. This can be seen clearly in, for example, the delay-based digital waveguide approach to physical modelling (Smith 1992), and in finite difference models (Bilbao 2009).

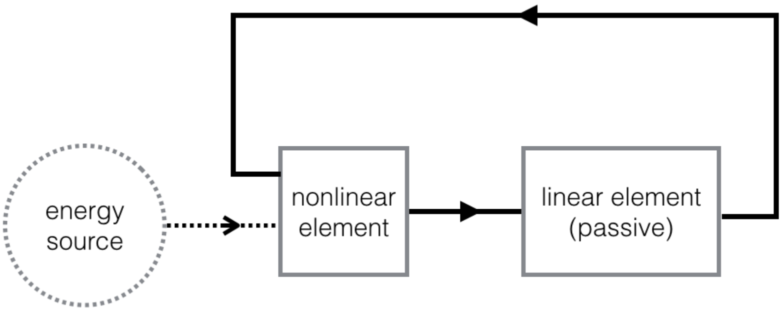

Fig. 1.
An idealised block diagram description of musical instruments as a coupling of a nonlinear element coupled with a passive linear element, as exemplified by brass, woodwind or bowed instruments, after McIntyre, Schumacher, and Woodhouse (1983).

The Gutter Synthesis process described in the next section is based largely on chaotic synthesis, but draws on elements of physical modelling—particularly the simplified model of an instrument shown in

[3] Although see Bilbao (2014) for further nonlinear behaviours in these "linear" elements

Fig. 1—in order to permit a continuum between explicitly electronic sounding noises, and outputs that sound plausibly acoustic, resembling physical situations such as rattling plates or scraped strings.

3 OVERVIEW OF THE SYNTHESIS PROCESS

The synthesis algorithm is based around a discrete map of the damped forced Duffing Oscillator. This is a relatively well understood nonlinear dynamical system (see Ott, Sauer, and Yorke (1994), Thompson and Stewart (2002), and Ueda (1980) for example). In the Gutter Synthesis implementation, the oscillator is coupled with a set of up to 24 bandpass filters. This makes the system resemble the simplified block diagram of musical instruments presented shown in Fig. 1: a nonlinear element is coupled with a linear passive element, and driven by an external energy source (akin to the forcing term in the Duffing oscillator in this instance). The Gutter synthesis software connects together eight of these coupled resonant Duffing voices into a dynamic network. Each voice is created as a Java object instantiated in MaxMSP inside the mxj~ object. The Duffing oscillator is described in more detail below, followed by the coupling with the bandpass filters, and an overview of the network.

3.1 The Damped Forced Duffing Oscillator

Notably for this work, the Duffing Oscillator is already a physical model in that it models a rigid beam that is driven by an external oscillating force (Thompson and Stewart 2002). The system is usually constructed as follows (Guckenheimer and Holmes 1983):

$$\ddot{x} + k\dot{x} + \alpha x^3 = Bcos(\omega t) \tag{1}$$

where k, α, B and ω are potentially controllable coefficients, and t is continuous time for the forcing term Bcos (ωt). This can be rendered as a discrete map:

$$\begin{aligned} x_{n+1} &= y_n \\ y_{n+1} &= -ky_n - \alpha x_n^3 - Bcos(\omega T_n) \end{aligned} \tag{2}$$

where Tn ∈ Z+. Direct sonifications of the Duffing system have been explored by Degazio (1993) and Dunn (2007), and it has been used as a control for synthesis parameters by Spasov (2015). Despite the relative simplicity, the system exhibits a range of complex phenomena around bifurcation points (Lakshmanan and Rajasekar 2003).

3.2 Coupled Resonance

The coupling with the resonance acts as a function on x_n in the discrete map. Each term x_n is run as an input to the set of bandpass filters, the output of which, $f(x_n)$, is used in place of x_n in Equation (2):

$$\begin{aligned} x_{n+1} &= y_n \\ y_{n+1} &= -ky_n - \alpha f(x_n)^3 - Bcos(\omega T) \end{aligned} \tag{3}$$

This can be seen in Figure 2. These filters effectively act as constraints on the Duffing oscillator, supporting oscillations at certain frequencies at the expense of others in a manner analogous to the regulating of a reed's behaviour by the impedance spectrum of an instrument's bore. With higher resonance settings, the filters can effectively prevent noisier behaviours, forcing the oscillations into the specific frequencies present in the filters. Decreasing the resonance allows for more chaotic regimes to develop, unconstrained by the filters. The result is a combination of the complexity of behaviour found in the raw nonlinear dynamical system with the more resonant and potentially more harmonic aspects of the filters.

The output of the filters is further constrained through a controllable lowpass filter that can optionally prevent the more abrasive high frequency content, and an arctan limiter to artificially constrain the output into the range ±1.

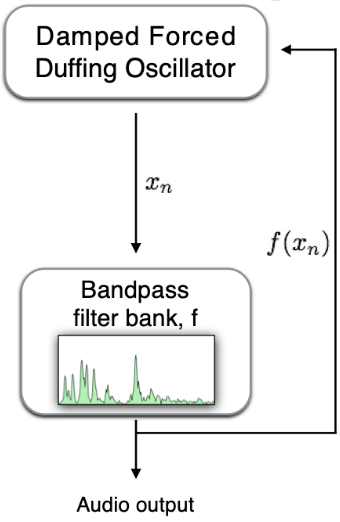

Fig. 2.
Each voice in the Gutter Synthesis software is a Duffing oscillator coupled to a set of bandpass filters.

3.3 Interactions Between Multiple Oscillators

The description given above describes an individual synthesis voice in the Gutter Synthesis program. A key aspect of the project is the coupling of up to eight voices. This is achieved by routing the output of each voice to the damping parameter, k in Equation (3), of every other voice at audio rate. This connection can be scaled to increase or decrease the amount of influence that each voice has on all other voices, but the parameter is constrained: $0.0001 \leq k \leq 1$. The constraint means that the audio signal—which can be positive or negative—will cause the damping to fluctuate.

A 2000 sample delay is placed in between each voice output and the damping inputs of each other oscillator. This lag appears to help the system as a whole to keep undulating, rather than quickly stabilising to a particular state.

4. CONTROL AND INTERACTION

This section looks at specific points of interest in interactions with the synthesis process. The parameters available in controlling the system are established, showing how they can be used to interact with the nonlinear dynamical system in different ways. The text descriptions of the interactions and the system's behaviour are supported by five video examples that can be found at vimeo.com/album/5707465. Interactions are examined first in relation to a single synthesis voice, then in relation to the interlinked network of eight voices.

The user interface for the synthesiser is shown in Fig. 3. The controls apply to all eight voices (a separate control panel can be used to alter the voices individually). These parameters relate to the equations above as follows:

- the *gain* parameter is inside the resonant Duffing loop, scaling the value of $f(x_n)$ in Eq. 3 rather than an external gain, and therefore affects more than just the level of the output;
- the *damping* parameter is k in Eq. 3, as noted earlier;
- *mod* is the driving amplitude, B;
- *rate* alters the driving frequency, ω;
- Q controls the resonance of the bandpass filters;
- *soften* controls the smoothing for the lowpass filter within the resonant Duffing loop;
- *oscillator interaction* alters the scaling of the connections between each voices output and the damping value for all other voices;
- *pitch shift* scales the frequencies of all bandpass filters.

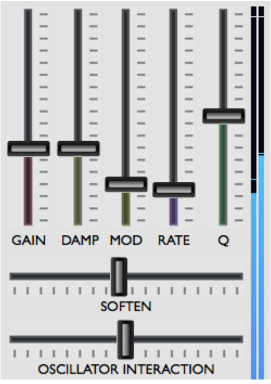

Fig. 3.
The interface for the Gutter Synthesis software.

4.1 Hysteresis and harmonic hopping

A key aspect of the interaction with chaotic systems is the potential for hysteresis (Mudd et al. 2015). The current state of the inputs to the system is not sufficient to determine the sonic behaviour. The prior state of the system, and hence the prior input to the system also plays an important role. This can be seen in the use of a single synthesis voice. Consider the system as represented in Equation (3) with the modulation, B, set to zero (an unforced Duffing oscillator). With the gain, resonance (Q) and modulation fixed at particular values, decreasing the damping tends to destabilise the perceived pitch being produced such that the system may snap to a different frequency, generally jumping to a different resonant frequency in the filter bank. The damping can then often be returned to its original value whilst retaining the new dominant frequency. This is shown in video number 2 in the accompanying online material. This "harmonic hopping" can also be heard with variations in other parameters such as resonance and gain (videos 3 and 4). This kind of interaction can be thought through visually in relation to the butterfly-like patterns created by the Lorenz attractor: if the user has real time control of the system parameters, they can find the boundary at which the trajectories settle into one "wing" or another of the butterfly, and cross and re-cross this boundary to hop from one orbit to the other (Mudd, Holland, and Mulholland 2019).

4.2 Constraining chaos with the resonant filter bank

With the resonance parameter at its minimum, the noisier tendencies of the raw Duffing oscillator can come through unconstrained. This is shown in video number 3 for a single synthesis voice, where the resonance parameter is increased from the minimum value until a stable pitch is produced. The system undergoes a series of seemingly discontinuous changes, moving from pulsed clicks through broadband noise bursts towards a stable low pitch, which rises slightly as the resonance approaches the maximum. The filter resonance becomes a useful way of constraining the more unpredictable aspects of the system: higher resonance values provide much more stable pitch outputs. The parameter can also be used to hop harmonics as noted above, where the parameter is lowered until the system hits a less stable point, a different tone emerges (usually closely related to the frequencies present in the filter bank) and the parameter can then be increased again to stabilize this new tone.

4.3 Holistic mapping

Chaotic synthesis processes and physical modelling present examples of a holistic mapping processes, as described by Hunt and Kirk (2000). Individual perceptual aspects are rarely controlled by individual parameters (e.g. separate controls for pitch, volume, brightness, etc.). This is demonstrated in video number 4 which shows how a single synthesis voice undergoes a range of transitions as the gain parameter is increased and decreased. To begin with the gain behaves almost like a volume control. As it is increased further, the tone starts to distort as other frequencies appear, with the tone eventually moving away completely from the original frequency to land more prominently on other harmonics. A range of unstable behaviours emerge for very high values, with rhythmic fluctuations in the presence of different harmonics. Figure 4 shows how the system can fluctuate by itself while the input parameters are left unchanged.

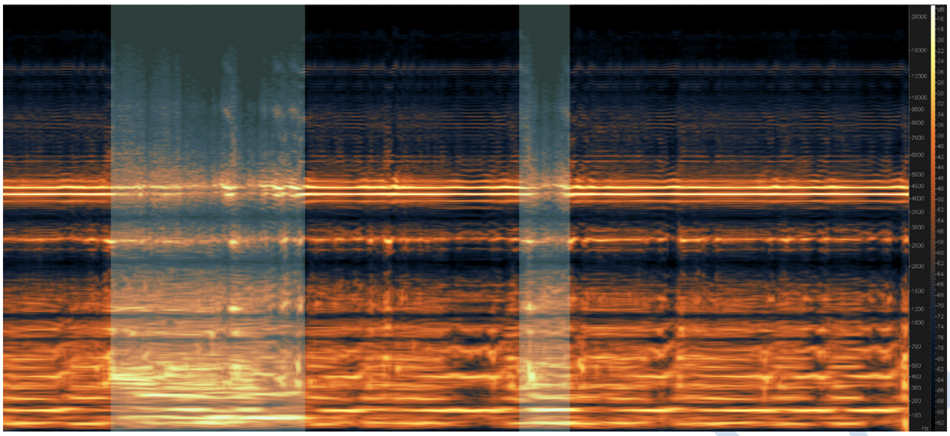

Fig. 4.
A sonogram showing four seconds from example video number 4 (1’17 to 1’21) where the parameters are unchanged. A range of frequencies are present, and the system can be seen to fluctuate between clear higher frequencies, and lower frequency rumbles (the latter are indicated by the blue shading).

4.4 Oscillator interactions

Video number 5 shows how the oscillator interaction processes change the behaviour of two synthesis voices. The two voices have different filter banks with different sets of frequencies (randomized logarithmically between 50 and 2000 Hz). The two voices are initially relatively stable, with distinct pitch identities as shown in Figure 5. The oscillator interaction parameter is increased from minimum to maximum (shown with the blue shading), which increases the extent to which the audio output from each voice affects the damping parameter of the other voice, as described at the start of this section. Three full sweeps of the parameter are shown in Figure 5, alongside the spectrogram of the two oscillators.

A first observation is—as can clearly be heard in the video and seen in the image—the response tends to be noisier as the interaction parameter is increased, with bands of noise around the resonant frequencies. Secondly, as with the variation in damping described above, the synthesis voices can be seen to make abrupt changes at two points. As Figure 5 shows, this is not an instant response to the particular value of the interaction parameter. The first jump is made as the parameter is held at ≈95%, with a switch in the key frequencies. Thirdly, it is somewhat surprising that there are *two* jumps. The first jump at around 45” is not a final resting place for the oscillators. When it is returned to 100% for the second time at around the 1’09 mark, both voices make a further jump to stabilize around much lower frequencies. These are maintained throughout the third sweep, with no discontinuous changes occurring.

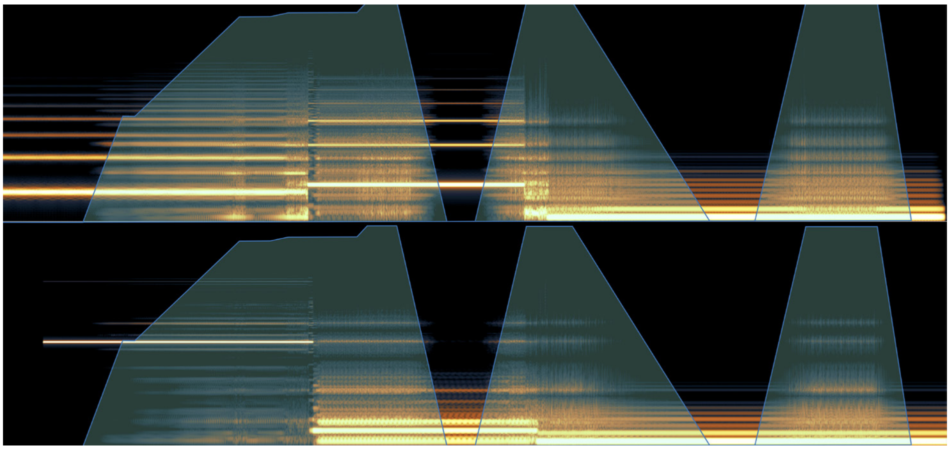

Fig. 5.
A sonogram of video number 5 (from 0'00 to 2'00) showing how two oscillators respond as the "oscillator interaction" parameter is varied. Three staggered sweeps are shown from 0-100% of the oscillator interaction parameter, indicated by the blue line.

5. DISCUSSION

Interactions with chaotic synthesis processes are often likened to collaboration with separate agents (Burns and Burtner 2004; Clutterbuck, Mudd, and Sanfilippo 2016), a refrain that can be also be found in relation to acoustic instruments in free improvisation (Borgo 2013; Lewis 2017; Unami 2005) and to hardware electronic sound making devices, as Keep notes in particular in relation to David Tudor (Keep 2009). In tracing his concept of *instrumentalization*, Keep highlights the importance of "bastardization"[4], pushing systems to do things they weren't made to do, and finding "fruitful edge-boundaries of unstable sonic activity". It is not always so straightforward to find these kinds of unstable boundaries in digital interactions however; the interface can only be bastardised so far. Chaotic synthesis processes provide one such way of realising this kind of interaction in digital contexts. In particular, the complex behaviours that can be found close to and across bifurcation points in chaotic systems appear to support this kind of edge-based interaction (Mudd, Holland, and Mulholland 2019).

The interactions traced above demonstrate this in relation to the Gutter Synthesis software, highlighting regions where unexpected things can happen, that neither the software designer nor the user can reliably predict. The set of possible outputs is not a sum of the possible inputs. Hysteresis permits even a single parameter to be explored almost endlessly, as what matters is not only the value of the parameter, but the current state of the system, and hence the history and timing of the user's input. Even a relatively low-resolution, single dimension of input may then be a source of considerable exploration, as shown in relation to the gain parameter in section 4.3 or the interaction parameter in section 4.4. When the system jumps to a new state, the nature of how it responds to different values of the input parameters can be very different. The fact that the system may take some time to transition to a new state (as shown in section 4.4) highlights the importance of timing and of rate-of-change in the user input. Moving a slider from point A to point B can yield very different results depending on how fast it was done, how long it was left in different regions in between and so on. The sound being made at point B could be radically

4. A term borrowed from John Richards.

different if the movement was very gradual, compared to the same movement performed very rapidly.

The link between chaotic synthesis and physical modelling demonstrates a connection between these kinds of digital interactions and real-world interactions encountered in acoustic musical instruments. Indeed, some of the kinds of interactions found in acoustic instruments may be down to their potential for chaotic processes. The same kind of hysteresis discussed above in sections 4.1, 4.2 and 4.3 can be found in interactions with wind instruments (McIntyre, Schumacher, and Woodhouse 1983) and bowed strings (Fletcher 1999) for instance. Considering the role that chaotic processes and nonlinear interactions play in other domains may also be instructive. Paint and paintbrushes afford similar kinds of interactions: both the tool and the medium exhibit hysteresis in different ways: the bristles can be considered as a set of interconnected springs (Chu and Tai 2002), and the paint itself alters its behaviour as it dries (Baxter, Wendt, and Lin 2004; Chhabra 2010). As with instrumentalizing approaches to music, the complexity of interaction appears to afford particular kinds of creative enquiry and exploration that engage with these complexities.

6. CONCLUSIONS

This paper has introduced the Gutter Synthesis project as a linking point between chaotic synthesis on the one hand and physical modelling on the other. Specific interactions with the synthesis processes have been considered that highlight the role of hysteresis in exploratory engagements. Chaotic synthesis processes in general were linked to the edge-like or boundary-based interactions that Keep describes in relation to his concept of instrumentalizing: drawing creative inspiration from the specific properties of sonic objects. Boundary points in interactions with the Gutter Synthesis process were examined, along with the rich and complex behaviours that can be found around these boundaries. The time-based nature of interactions with systems that exhibit hysteresis is considered, and put forward as a useful method for setting up digital interactions that can be fruitfully explored, even with a small number of low-resolution inputs.

REFERENCES

Baxter, William, Jeremy Wendt, and Ming C. Lin.
2004. “IMPaSTo: A Realistic, Interactive Model for Paint.” In *Proc. 3rd International Symposium on Non-Photorealistic Animation and Rendering* (NPAR’04), , 45–56.

Bilbao, Stefan.
2009. *Numerical Sound Synthesis.* John Wiley and Sons, Ltd.

Bilbao, Stefan.
2014. “The Changing Picture of Nonlinearity in Musical Instruments: Modeling and Simulation.” In *Proceedings of the International Symposium on Musical Acoustics*,.

Borgo, David.
2013. “The Ghost in the Music, or the Perspective of an Improvising Ant.” In *The Oxford Handbook of Critical Improvisation Studies, Volume 1*, eds. George E. Lewis and Benjamin Piekut. Oxford University Press.

Burns, Christopher, and Matthew Burtner.
2004. “Recursive Audio Systems: Acoustic Feedback in Composition.” *Leonardo Electronic Almanac* 12(2).

Chu, Nelson SH, and Chiew-Lan Tai.
2002. “An Efficient Brush Model for Physically-Based 3D Painting.” In *Proc. 10th Pacific Conference on Computer Graphics and Applications*, IEEE, , 413–424.

Degazio, Bruno.
1993. “Towards a Chaotic Musical Instrument.” In *Proceedings of the 1993 International Computer Music Conference*,, 393–395.

Dunn, David.
2007. *Autonomous and Dynamical Systems.*

Guckenheimer, John, and Philip Holmes.
1983. *Nonlinear Oscillations, Dynamical Systems, and Bifurcations of Vector Fields*. Springer.

Keep, Andy.
2009. “Improvising with Sounding Objects in Experimental Music.” In *The Ashgate Research Companion to Experimental Music*, Ashgate Publishing Limited, 113 – 130.

Lakshmanan, M., and S. Rajasekar.
2003. *Nonlinear Dynamics: Integrability, Chaos, and Patterns*. Springer.

Mackenzie, J. P.
1995. “Chaotic Predictive Modelling of Sound.” In *Proceedings of the 1995 International Computer Music Conference*, , 49–56.

Mudd, Tom.
2017. “Nonlinear Dynamics in Musical Interactions.” PhD Thesis. Centre for Research in Computing, The Open University.

Mudd, Tom, Simon Holland, and Paul Mulholland.
2019. “Nonlinear Dynamical Processes in Musical Interactions: Investigating the Role of Nonlinear Dynamics in Supporting Surprise and Exploration in Interactions with Digital Musical Instruments.” *International Journal of Human-Computer Studies* 128: 27–40.

Ott, Edward, Tim Sauer, and James A. Yorke.
1994. Coping with Chaos : *Analysis of Chaotic Data and the Exploitation of Chaotic Systems*. John Wiley and Sons Inc.

Chhabra, Rajendra P.
2010. “Non-Newtonian Fluids: An Introduction.” *In Rheology of Complex Fluids*, eds. J. Murali Krishnan, Abhijit P. Deshpande, and P. B. Sunil Kumar. Springer.

Clutterbuck, Tristan, Tom Mudd, and Dario Sanfilippo.
2016. “A Practical and Theoretical Introduction to Chaotic Musical Systems.” In *Proc. of the International Conference on Live Interfaces*, eds. Thor Magnusson, Chris Kiefer, and Sam Duffy. Experimental Music Technologies (EMuTe) Lab, University of Sussex.

Dobson, Richard, and John Fitch.
1995. “Experiments with Chaotic Oscillators.” In *Proceedings of the 1995 International Computer Music Conference*,, 45–48.

Fletcher, Neville H.
1999. “The Nonlinear Physics of Musical Instruments.” *Reports on Progress in Physics* 62: 723–764.

Hunt, Andy, and Ross Kirk.
2000. “Mapping Strategies for Musical Performance.” In *Trends in Gestural Control of Music*, eds. M. Wanderley and eds. M. Battier. Ircam - Centre Pompidou, 231–258.

Kuivilla, Ronald.
2004. “Open Sources: Words, Circuits and the Notation-Realization Relation in the Music of David Tudor.” *Leonardo Music Journal* 14.

Lewis, George E.
2017. “From Network Bands to Ubiquitous Computing.” In *Improvisation and Social Aesthetics*, eds. Georgina Born, Eric Lewis, and Will Straw. Duke University Press, 91–111.

McIntyre, M. E., R. T. Schumacher, and J. Woodhouse.
1983. “On the Oscillations of Musical Instruments.” *Journal of the Acoustical Society of America* 74(5): 1325–1345.

Mudd, Tom.
2018. *Gutter Synthesis (CD + Software).*
Antwerp: Entr’Acte.

Mudd, Tom, Simon Holland, Paul Mulholland, and Nick Dalton.
2015. “Investigating The Effects Of Introducing Nonlinear Dynamical Processes Into Digital Musical Interfaces.” In *Proceedings of Sound and Music Computing Conference* 2015, Maynooth,.

Pressing, Jeff.
1988. “Nonlinear Maps as Generators of Musical Design.” *Computer Music Journal* 12(2): 35–46.

Radunskaya, Ami.
1996. “Chaos and Non-Linear Models.” In *Proceedings of the 1996 International Computer Music Conference*, , 440–443.

Scipio, Agostino Di.
1990. “Composition by Exploration of Non-Linear Dynamical Systems.” In *Proceedings of the 1990 International Computer Music Conference*, , 324–327.

Smith, Julius O.
1992. “Physical Modeling Using Digital Waveguides.” *Computer Music Journal* 16: 74–91.

Spasov, Miroslav.
2015. “Using Strange Attractors to Control Sound Processing in Live Electroacoustic Composition.” Computer Music Journal 39(3): 25–45.

Thompson, J. M. T., and H. B. Stewart.
2002. *Nonlinear Dynamics and Chaos*. John Wiley and Sons.

Truax, Barry.
1990. “Chaotic Non-Linear Systems and Digital Synthesis: An Exploratory Study.” In *Proceedings of the 1990 International Computer Music Conference*, , 100–103.

Ueda, Yoshisuke.
1980. *Steady Motions Exhibited by Duffing’s Equation - A Picture Book of Regular and Chaotic Motions*. Nagoya University.

Unami, Taku.
2005. “What Are You Doing with Your Music?” In *Blocks of Consciousness and the Unbroken Continuum*, eds. Brian Marley and Mark Wastell. Sound 323.

Välimäki, Vesa, Jyri Pakarinen, Cumhur Erkut, and Matti Karjalainen.
2006. “Discrete-Time Modelling of Musical Instruments.” *Reports on Progress in Physics* 69: 1–78.